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ENERGY DISSIPATION DURING VIBRATIONAL MOTION IN A LIQUID

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The paper reports calculation results and an approximate description of the power expended by a system in vibrational motion in a medium with drag proportional to velocity. It is shown that the power expended in vibrational motion is proportional to the squared mean velocity and the proportionality coefficient depends only on the drag of the medium.

Introduction. The mean velocity of so-called vibrational motion [1, 2] is not a single parameter that describes this type of motion. Currently available results [3–5] are of a particular nature and cannot be used to evaluate, for example, the energy loss during vibrational motion in a resistant medium [6, 7]. In some cases, this characteristic can be even more important than the mean velocity of vibrational motion. Knowing the energy loss expended by a system in vibrational motion and mean velocity, it is possible not only to adequately describe vibrational motion but also to determine optimal conditions for the most effective process.

Energy Loss During Vibrational Displacement. The motion direction is collinear to only two forces: the internal force $\boldsymbol{F}$ exerted by an unbalanced body of mass m on a platform of mass M, and the drag force of the medium $\boldsymbol{F}_r$, whose value is proportional to the velocity of motion of the platform $\boldsymbol{v}$ relative to the liquid L (Fig. 1). Over the time $t_0 < t < t^0$ reckoned from the beginning of motion t_0 to the moment of stop t^0 and equal to the period T of vibrations of the body of mass m, these forces perform zero work A,

$$A = \int_{t_0}^{t^0} (\boldsymbol{F} + \boldsymbol{F}_r)\boldsymbol{v}\,dt = 0,$$

because the motion of the platform is described by the equation

$$M\,\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F} + \boldsymbol{F}_r.$$

This implies that the work performed by the internal force $\boldsymbol{F}$ is completely expended in overcoming the drag of the medium and to estimate the energy losses, it suffices to calculate the mean power

$$P_r = \frac{1}{T}\int_{t_0}^{t^0} \lambda v^2\,dt$$

expended by the system in vibrational motion. Here λ is the drag coefficient of the media, which has different values for different directions of motion of the platform:

$$\lambda = \begin{cases} \lambda_+ & \text{for} \quad v > 0, \\ \lambda_- & \text{for} \quad v < 0. \end{cases}$$

The velocity of the platform v is a function of m, M, T, λ_+, λ_-, and the amplitudes of oscillations of the unbalanced body a and is defined by solving the following equation [7]:

$$\frac{d\vartheta}{d\tau} + \Big(\frac{1-\delta}{2}\,\mathrm{sign}\,(\vartheta) + \frac{1+\delta}{2}\Big)\vartheta - \frac{1}{\theta^2}\cos\frac{2\pi\tau}{\theta} = 0. \tag{1}$$

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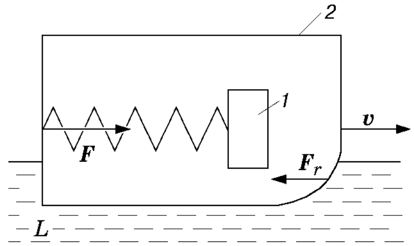

Fig. 1. Diagram of vibrational motion of a platform in a liquid:
1) unbalanced body; 2) platform.

Here $\vartheta = M_0^2 v/(4\pi^2 ma\lambda_+)$, $\delta = \lambda_-/\lambda_+$, $\tau = \lambda_+ t/M_0$, $\theta = \lambda_+ T/M_0$, and $M_0 = m + M$. In these variables, the normalized power

$$\rho = M_0^4 P_r/(16\pi^4 m^2 a^2 \lambda_+^3)$$

depends only on the variables θ and δ and is defined by the expression

$$\rho = \frac{1}{\theta}\left(\int\limits_{\tau_0}^{\tau_1} \vartheta^2\, d\tau + \delta \int\limits_{\tau_1}^{\tau^0} \vartheta^2\, d\tau\right), \tag{2}$$

where τ_0, τ_1, and τ^0 are the values of the variable τ at t equal to t_0, t_1, and t^0, respectively and t_1 is the time of an intermediate stop of the platform: $t_0 < t_1 < t_0 + T$.

The procedure of solving the problem consists of numerical integration of the equation of motion (1), determination of the time t_1 when the velocity of the platform becomes equal to zero, calculation of the normalized power (2), and processing of the calculation results. The part of the problem that includes calculations does not involve great difficulties. In the calculations, it is necessary to chose the value t_0 corresponding to steady motion, for example, $t_0 = 10T$. In this case, $t^0 = t_0 + T$. Problems arise in interpreting the calculations of the power expended by the system in vibrational motion. The calculation results obtained can be used to optimize the regime of vibrational motion. Use of the dependence of the normalized power ρ on the asymmetry parameter δ and the normalized period of vibrations θ is ineffective in this case. Therefore, it is reasonable to present result in universal form. In particular, this can be done using the self-similar representation of [7], which is a consequence of the symmetry of the system parameters with change in the direction of motion. For example, the value of the variable $\theta_\delta = \theta/(1 + 1/\delta)$ does not change when λ_+ and λ_- are replaced simultaneously by λ_- and λ_+, respectively. In fact, this replacement is identical to the transformation

$$\lambda_+ \to \lambda_+\lambda_-/(\lambda_+ + \lambda_-). \tag{3}$$

In contrast to the velocity [7], the sign and value of the power do not change with change in the direction of motion. This implies that the normalized power should be converted by the rule $\rho \to \rho(1 + 1/\delta)^3$ because the power is determined not by the velocity of the platform but by the squared velocity, to which a symmetric transformation should correspond with change in the direction of motion.

The aforesaid can be used for approximate description of calculation results, which is not always valid [8]. In our case, this approach gives fairly accurate results for all $\theta > 0.1$ and any asymmetry parameters, including $\delta = 1$ (Fig. 2). At the same time, this representation is quite definite. For specified values of θ_δ and $\rho_\delta = \rho(1 + 1/\delta)^3$, it allows one not only to find the energy expended in vibrational motion but also to determine the normalized period θ.

As the vibration period increases, the expended power tends rapidly to zero, which is confirmed by the approximate analytical dependence of the normalized power ρ on the normalized period θ:

$$\rho \approx \delta(\delta + 1)/\{5\theta^2[\theta^2\delta^2 + 7(\delta + 1)^2]\}. \tag{4}$$

The results of numerical calculations presented in Fig. 2 and the approximate relation (4) show that the power P_r is not zero even in the case where the infinite (unlimited) displacement of the platform is absent, i.e., for $\delta = 1$. In the absence of asymmetry, the center of gravity of the system of bodies is at rest although the platform performs forced nonharmonic oscillations about the medium.

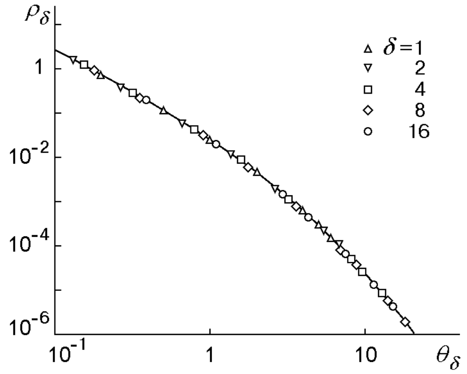

Fig. 2. Self-similar dependence of the normalized power $\rho_\delta = \rho(1+1/\delta)^3$ expended by the system in vibrational motion on the normalized vibration period $\theta_\delta = \theta/(1+1/\delta)$ and the asymmetry parameter δ: solid curve refers to relation (4) and the points refer to results of numerical calculation of the normalized power at $0.1 \leqslant \theta \leqslant 16$.

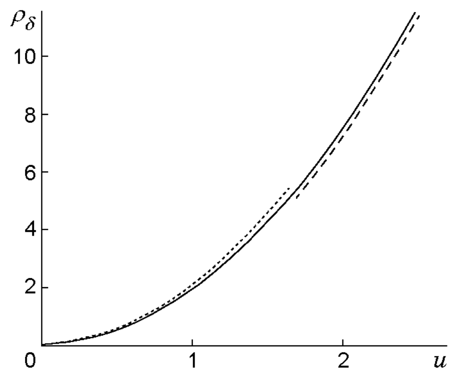

Fig. 3. Curve of normalized power versus mean normalized velocity (solid curve) and asymptotic relations (6) and (7) for large ($u \to \infty$) (dashed curve) and small ($u \to 0$) (dotted curve) velocities of vibrational motion.

Energy Loss and Mean Velocity. The energy lost by the system tends rapidly to zero, with increase in vibration period. To explain this phenomenon, it is reasonable to establish the relationship between the power expended in vibrational motion and the mean velocity. For this, from the equation defining the mean normalized velocity $\bar{\vartheta}$ [7], it is necessary to find the normalized period

$$\theta \approx (\delta - 1)/\{[64\bar{\vartheta}(\delta+1)^2 + 25\delta(\delta-1)/8]^{1/2}\bar{\vartheta}^{1/2}\}$$

and substitute it into expression (4) for the normalized power. As a result, we obtain

$$\rho = \delta^3(512u + 25)^2 u^2/\{40(\delta+1)^3[8 + 7u(512u + 25)]\}, \qquad (5)$$

where $u = \bar{\vartheta}(\delta+1)^2/[\delta(\delta-1)]$ is the normalized mean velocity written with the argument $\theta/(1+1/\delta)$ eliminated. For large values of u, expression (5) reduces to the simple form

$$P_r \approx (64/35)\lambda' \bar{v}^2, \qquad (6)$$

where $\lambda' = \lambda_+\lambda_-(\lambda_- + \lambda_+)/(\lambda_- - \lambda_+)^2$.

A similar relation is observed for low velocities:

$$P_r \approx (125/64)\lambda' \bar{v}^2. \qquad (7)$$

Figure 3 shows a curve of $\rho_\delta = \rho(1+1/\delta)^3$ versus u and the asymptotic relations (6) and (7). The fact that over the entire velocity range, the power loss is proportional to the squared mean velocity is not unexpected because when the body moves without vibration in a medium with drag proportional to the velocity of the body v, the expended power is $P = \lambda v^2$. The proportionality coefficient in expressions (6) and (7) do not depend on parameters of the system, Therefore, it is impossible to determine the conditions corresponding to the minimum energy loss for a specified mean velocity of vibrational motion. The optimal regime of vibrational motion can be established only after adopting additional conditions.

REFERENCES

1. I. I. Blekhman, *Vibrational Mechanics* [in Russian], Nauka, Moscow (1994).
2. R. F. Nagaev, *Periodic Regimes of Vibrational Motion* [in Russian], Nauka, Moscow (1978).
3. R. S. Sharmat, "Analysis of the vibra-lo," *Mech. Mach. Theory*, **13**, No. 2, 208–212 (1978).
4. A. A. Kobrinskii, "Dynamics of a vibration engine," *Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela*, No. 2, 42–47 (1976).
5. S. A. Gerasimov, "A real model of vibratory-impact propulsive device," *Probl. Appl. Phys.*, **6**, 117–118 (2000).
6. R. F. Nagaev and E. A. Tamm, "Vibrational motion in a medium with quadratic drag," *Mashinovedenie*, No. 4, 3–8 (1980).
7. S. A. Gerasimov, "Self-similarity of vibrational motion in a resistant medium," *J. Appl. Mech. Tech. Phys.*, **43**, No. 1, 90–92 (2002).
8. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics* [in Russian], Nauka, Moscow (1967).